Measuring and generating signals with ADC's and DAC's

1) Terms used

Full Scale Range, Least Significant Bit (LSB), Resolution, Linearity, Accuracy, Gain Error, Offset, Monotonicity, Conversion time, Settling time.

2) Quantization Error and resultant Signal - to - Noise Ratio

Suppose that the instantaneous value of the input voltage is measured by an ADC with a Full scale range of V_{fs} volts, and a resolution of n bits. The real value can change through a range of $q = V_{fs} / 2^n$ volts without a change in measured value occurring.

It follows that the value of the measured signal is $V_m = V_s \pm e$,

where

V_m is the measured value,

V_s is the actual value, and

e is the error.

The maximum value of error in the measured signal is

emax = $(^1/_2)(V_{fs} / 2^n)$ or emax = $^q/_2$ since $q = V_{fs} / 2^n$

(Assuming that the measured value represents the value at the centre of the measurement band).

The RMS value of quantization error voltage is

$$e_{qe} = \sqrt{\left(\frac{1}{q}\right)\int_{-q/2}^{+q/2} e^2\, de} \qquad \text{whence} \qquad \underline{e_{qe} = q / 2\sqrt{3} \quad \text{volts rms}}$$

The Signal to Noise Ratio (SNR) is defined as

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}}$$

It is normally quoted on a logarithmic scale, in deciBels (dB).

$$SNR_{dB} = 10\log_{10}\frac{\text{Signal Power}}{\text{Noise Power}} \qquad \text{or} \qquad SNR_{dB} = 20\log_{10}\frac{\text{RMS Signal Voltage}}{\text{RMS Noise Voltage}}$$

In this case if a sinusoidal input signal is matched to the Full Scale Range of the converter, then the peak - to - peak value of the signal is V_{fs}. The RMS signal voltage is then

$V_{in(RMS)} = V_{fs} / 2\sqrt{2}$ volts RMS

The error, or quantization noise signal is $e_{qe} = q / 2\sqrt{3}$ volts RMS

Thus the signal - to - noise ratio in dB. is $SNR_{dB} = 20\log_{10}\left(V_{fs} / 2\sqrt{2}\right)/\left(q / 2\sqrt{3}\right)$

since Vfs = 2^n q, then $SNR_{dB} = 20\log_{10}\left(2^n q / 2\sqrt{2}\right)/\left(q / 2\sqrt{3}\right)$

which simplifies to $SNR_{dB} = 6.02n + 1.78$

N.B. This equation is true only if the input signal is exactly matched to the Full Scale Range of the converter. For signals whose amplitude is less than the FSR the Signal - to - Noise Ratio will be reduced.

Resolution and Signal:Noise Ratio for signals coded as n bits			
•	•	•	•
bits, n•	levels, 2^n•	• Weighting of LSB, 2^-n•	SNR, dB•
•	•	• •	•
1•	2•	• 0.5•	8•
2•	4•	• 0.25•	14•
3•	8•	• 0.125•	20•
4•	16•	• 0.0625•	26•
5•	32•	• 0.0313•	32•
6•	64•	• 0.0156•	38•
7•	128•	• 0.00781•	44•
8•	256•	• 0.00391•	50•
9•	512•	• 0.00195•	56•
10•	1024•	• 0.00098•	62•
11•	2048•	• 0.00049•	68•
12•	4096	0.00024	74
13•	8192•	• 0.000122•	80•
14•	16384•	• 0.000061•	86•
15•	32768•	• 0.000031•	92•
16•	65536•	• 0.000015•	98•
•	•	• •	•

3) Limit of resolution of measurements on a changing signal

Errors can arise if the voltage being measured changes significantly during the measurement period. The magnitude of the error that such a change causes depends on the process by which the conversion is carried out. Integrating converters such as dual-slope converters will produce a result which describes the average value at the input during the measurement interval; counter-ramp converters will indicate the input value at the end of the conversion period; whilst successive approximation converters can give a measurement which is in error by the total amount of change during the conversion period.
It follows that the rate at which the input signal changes imposes a limit on the resolution of the conversion process.
If we wish to achieve a maximum resolution of ± ½ LSB then the input signal must not change by more than ½ LSB during the conversion period.

If we suppose that we are measuring a sinusoidal waveform of amplitude 2 * Vpeak then
$v = V_{peak} \sin (2 \pi f t)$ and
$dv / dt = V_{peak} (2 \pi f) \cos (2 \pi f t)$
This is a maximum at t = 0, π, 2π, ... and has a value of
$(dv / dt)max = 2 \pi f V_{peak}$

During a conversion time of τ the input voltage will change by
$\delta v = 2 \pi f V_{peak} \tau$
We already know the limit of change in input voltage for maximum resolution is

$\delta v = V_{FS} / 2 * 2^n$, so we have

$V_{FS} / 2 * 2^n = 2 \pi f V_{peak} \tau$

This can be rearranged to give

$f = (V_{FS} / 2 * V_{peak}) * (1 / 2^n) * (1 / 2\pi\tau)$ or

$\tau = (V_{FS} / 2 * V_{peak}) * (1 / 2^n) * (1 / 2\pi f)$

In the same way, if we wish to evaluate the maximum resolution available for a particular converter at a particular frequency we can rearrange the equation to give

$R = (V_{FS} / 2 * V_{peak}) * (1 / 2^n) * (1 / 2\pi f\tau)$

where R is the resolution expressed in terms of number of LSB's.

Example

We have an 8 bit successive approximation converter with a conversion time of 10 microseconds. What is the maximum frequency sinewave which can be applied at the input if we are to preserve the measurement accuracy at ±½ LSB. Assume the sinewave input amplitude is matched to the FSR of the converter, i.e. $V_{FS} = 2 * V_{peak}$

$f = (V_{FS} / 2 * V_{peak}) * (1 / 2^n) * (1 / 2\pi\tau)$

$f = (1) * (1 / 2^8) * (1 / 2\pi * 10^{-5})$

$f = (1 / 256) * (10^5 / 2\pi)$

$f = 10^5 / (512 \pi) = 62$ Hz

How often should we measure an incoming signal?

The calculations above imply that if we are to accurately record a signal we must take 1,600 measurements within one period of the highest frequency present in the wave, which is clearly a very demanding task. The criterion we have applied is really only appropriate if we need to be able to determine with a single measurement the instantaneous value of an incoming signal to the limit of resolution of our converter.
An alternative argument is that we can reconstruct all the frequency information present in the signal if we take samples at a rate higher than twice the maximum frequency present.
(i.e. two samples per cycle of the highest frequency component)
This is called the "Nyquist criterion". This criterion is most appropriate if we are aquiring data from a modulated carrier signal.

We need a more generally appropriate criterion which will allow us to decide how often we should measure an incoming signal.
If we sample an incoming signal v = f(t) at regular intervals T with a converter having a conversion time of τ, the signal is modified by a factor
Venv(f) = (2τ / T)(sin πτf)/(πτf)
due to the transfer function of the sampling system.
(Reference: Kripps, M.   Microcomputer Interfacing)
this results in a measurement error given by
$\delta v / V_{peak}$ = 1 - ((sin πτf)/(πτf))
if we expand (sin x) using the MacLaurin expansion we get

$$\sin(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ . \ . \ .$$

hence $$\frac{\sin(x)}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \ . \ . \ .$$

and $$dv \big/ V_{peak} = \frac{(ptf)^2}{3!} - \frac{(ptf)^4}{5!} + \frac{(ptf)^6}{7!} - \ldots$$

Example
If we use a successive approximation ADC with a conversion time of 10 microseconds to take measurements on a sinusoidal signal at a frequency of 1000Hz. then the error introduced by the sampling function is

$$dv \big/ V_{peak} = \frac{(ptf)^2}{3!} - \frac{(ptf)^4}{5!} + \frac{(ptf)^6}{7!} - \ldots$$

and πτf = π * 10^{-5} * 1000   πτf = 3.14 * 10^{-2}

$\delta v / V_{peak}$ = (1.65 * 10^{-4}) - (8.1 * 10^{-9}) - ...

$\delta v / V_{peak}$ = 0.0165 %

Although it is not easy to solve this equation for f or τ, we find by experiment that if we use a conversion time which allows us to take twenty samples within one period of a wave, we obtain a measurement accuracy of 0.41% . This matches the resolution of an 8 bit converter.
Proof:
Let f = 1 kHz and τ = 50 microseconds.

Then $\pi\tau f = \pi * 5 * 10^{-5} * 1000$ and $\pi\tau f = 0.157$

$$dv/V_{peak} = \frac{(ptf)^2}{3!} - \frac{(ptf)^4}{5!} + \frac{(ptf)^6}{7!} - \ldots$$

$\delta v / V_{peak} = (4.112 * 10^{-3}) - (5.07 * 10^{-6}) + \ldots$

$\delta v / V_{peak} = 4.107 * 10^{-3}$ or 0.41%

It is interesting to see that at $2*\tau = 1/f$ (Nyquist)

$\pi\tau f = \pi * 0.5$

$\delta v / V_{peak} = 1 - ((\sin \pi\tau f)/(\pi\tau f))$

$\delta v / V_{peak} = 1 - ((\sin \pi/2)/(\pi/2))$

and thus $\delta v / V_{peak} = 0.363$

This shows that we cannot make accurate measurements of signal level at or near the Nyquist frequency.